



Credit Risk

Chapter 22

Credit Ratings

- In the S&P rating system, AAA is the best rating. After that comes AA, A, BBB, BB, B, CCC, CC, and C
- The corresponding Moody's ratings are Aaa, Aa, A, Baa, Ba, B, Caa, Ca, and C
- Bonds with ratings of BBB (or Baa) and above are considered to be "investment grade"

Historical Data

Historical data provided by rating agencies are also used to estimate the probability of default

Cumulative Ave Default Rates (%) (1970-2006, Moody's, Table 22.1, page 498)

	1	2	3	4	5	7	10
Aaa	0.000	0.000	0.000	0.026	0.099	0.251	0.521
Aa	0.008	0.019	0.042	0.106	0.177	0.343	0.522
A	0.021	0.095	0.220	0.344	0.472	0.759	1.287
Baa	0.181	0.506	0.930	1.434	1.938	2.959	4.637
Ba	1.205	3.219	5.568	7.958	10.215	14.005	19.118
B	5.236	11.296	17.043	22.054	26.794	34.771	43.343
Caa-C	19.476	30.494	39.717	46.904	52.622	59.938	69.178

Interpretation

- The table shows the probability of default for companies starting with a particular credit rating
- A company with an initial credit rating of Baa has a probability of 0.181% of defaulting by the end of the first year, 0.506% by the end of the second year, and so on

Do Default Probabilities Increase with Time?

- For a company that starts with a good credit rating default probabilities tend to increase with time
- For a company that starts with a poor credit rating default probabilities tend to decrease with time

Default Intensities vs Unconditional Default Probabilities (page 498-99)

- The default intensity (also called hazard rate) is the probability of default for a certain time period conditional on no earlier default
- The unconditional default probability is the probability of default for a certain time period as seen at time zero
- What are the default intensities and unconditional default probabilities for a Caa rate company in the third year?

Default Intensity (Hazard Rate)

- The default intensity (hazard rate) that is usually quoted is an instantaneous
- If $V(t)$ is the probability of a company surviving to time t

$$V(t + \Delta t) - V(t) = -\lambda(t)V(t)\Delta t$$

This leads to

$$V(t) = e^{-\int_0^t \lambda(s) ds}$$

The cumulative probability of default by time t is

$$Q(t) = 1 - V(t) = 1 - e^{-\int_0^t \lambda(s) ds}$$

Recovery Rate

The recovery rate for a bond is usually defined as the price of the bond immediately after default as a percent of its face value

Recovery Rates

(Moody's: 1982 to 2006, Table 22.2, page 499)

Class	Mean(%)
Senior Secured	54.44
Senior Unsecured	38.39
Senior Subordinated	32.85
Subordinated	31.61
J unior Subordinated	24.47

Estimating Default Probabilities

- Alternatives:
 - Use Bond Prices
 - Use CDS spreads
 - Use Historical Data
 - Use Merton's Model

Using Bond Prices (Equation 22.2, page 500)

Average default intensity over life of bond is approximately

$$\frac{s}{1 - R}$$

where s is the spread of the bond's yield over the risk-free rate and R is the recovery rate

More Exact Calculation

- Assume that a five year corporate bond pays a coupon of 6% per annum (semiannually). The yield is 7% with continuous compounding and the yield on a similar risk-free bond is 5% (with continuous compounding)
- Price of risk-free bond is 104.09; price of corporate bond is 95.34; expected loss from defaults is 8.75
- Suppose that the probability of default is Q per year and that defaults always happen half way through a year (immediately before a coupon payment).

Calculations (Table 22.3, page 501)

Time (yrs)	Def Prob	Recovery Amount	Risk-free Value	LGD	Discount Factor	PV of Exp Loss
0.5	Q	40	106.73	66.73	0.9753	$65.08Q$
1.5	Q	40	105.97	65.97	0.9277	$61.20Q$
2.5	Q	40	105.17	65.17	0.8825	$57.52Q$
3.5	Q	40	104.34	64.34	0.8395	$54.01Q$
4.5	Q	40	103.46	63.46	0.7985	$50.67Q$
Total						$288.48Q$

Calculations continued

- We set $288.48Q = 8.75$ to get $Q = 3.03\%$
- This analysis can be extended to allow defaults to take place more frequently
- With several bonds we can use more parameters to describe the default probability distribution

The Risk-Free Rate

- The risk-free rate when default probabilities are estimated is usually assumed to be the LIBOR/swap zero rate (or sometimes 10 bps below the LIBOR/swap rate)
- To get direct estimates of the spread of bond yields over swap rates we can look at asset swaps

Real World vs Risk-Neutral Default Probabilities

- The default probabilities backed out of bond prices or credit default swap spreads are risk-neutral default probabilities
- The default probabilities backed out of historical data are real-world default probabilities

A Comparison

- Calculate 7-year default intensities from the Moody's data (These are real world default probabilities)
- Use Merrill Lynch data to estimate average 7-year default intensities from bond prices (these are risk-neutral default intensities)
- Assume a risk-free rate equal to the 7-year swap rate minus 10 basis point

Real World vs Risk Neutral Default Probabilities, 7 year averages (Table 22.4, page 503)

Rating	Real-world default probability per yr (% per annum)	Risk-neutral default probability per yr (% per year)	Ratio	Difference
Aaa	0.04	0.60	16.7	0.56
Aa	0.05	0.74	14.6	0.68
A	0.11	1.16	10.5	1.04
Baa	0.43	2.13	5.0	1.71
Ba	2.16	4.67	2.2	2.54
B	6.10	7.97	1.3	1.98
Caa-C	13.07	18.16	1.4	5.50

Risk Premiums Earned By Bond Traders

(Table 22.5, page 504)

Rating	Bond Yield Spread over Treasuries (bps)	Spread of risk-free rate used by market over Treasuries (bps)	Spread to compensate for default rate in the real world (bps)	Extra Risk Premium (bps)
Aaa	78	42	2	34
Aa	87	42	4	42
A	112	42	7	63
Baa	170	42	26	102
Ba	323	42	129	151
B	521	42	366	112
Caa	1132	42	784	305

Possible Reasons for These Results

- Corporate bonds are relatively illiquid
- The subjective default probabilities of bond traders may be much higher than the estimates from Moody's historical data
- Bonds do not default independently of each other. This leads to systematic risk that cannot be diversified away.
- Bond returns are highly skewed with limited upside. The non-systematic risk is difficult to diversify away and may be priced by the market

Which World Should We Use?

- We should use risk-neutral estimates for valuing credit derivatives and estimating the present value of the cost of default
- We should use real world estimates for calculating credit VaR and scenario analysis

Merton's Model (page 506-507)

- Merton's model regards the equity as an option on the assets of the firm
- In a simple situation the equity value is

$$\max(V_T - D, 0)$$

where V_T is the value of the firm and D is the debt repayment required

Equity vs. Assets

An option pricing model enables the value of the firm's equity today, E_0 , to be related to the value of its assets today, V_0 , and the volatility of its

$$E_0 = V_0 N(d_1) - De^{-rT} N(d_2)$$

where

$$d_1 = \frac{\ln(V_0/D) + (r + \sigma_V^2/2)T}{\sigma_V \sqrt{T}}; \quad d_2 = d_1 - \sigma_V \sqrt{T}$$

Volatilities

$$\sigma_E E_0 = \frac{\partial E}{\partial V} \sigma_V V_0 = N(d_1) \sigma_V V_0$$

This equation together with the option pricing relationship enables V_0 and σ_V to be determined from E_0 and σ_E

Example

- A company's equity is \$3 million and the volatility of the equity is 80%
- The risk-free rate is 5%, the debt is \$10 million and time to debt maturity is 1 year
- Solving the two equations yields $V_0=12.40$ and $\sigma_v=21.23\%$

Example continued

- The probability of default is $N(-d_2)$ or 12.7%
- The market value of the debt is 9.40
- The present value of the promised payment is 9.51
- The expected loss is about 1.2%
- The recovery rate is 91%

The Implementation of Merton's Model

- Choose time horizon
- Calculate cumulative obligations to time horizon. This is termed by KMV the “default point”. We denote it by D
- Use Merton's model to calculate a theoretical probability of default
- Use historical data or bond data to develop a one-to-one mapping of theoretical probability into either real-world or risk-neutral probability of default.

Credit Risk in Derivatives Transactions (page 510-512)

Three cases

- Contract always an asset
- Contract always a liability
- Contract can be an asset or a liability

General Result

- Assume that default probability is independent of the value of the derivative
- Consider times $t_1, t_2, \dots, t_n$ and default probability is q_i at time t_i . The value of the contract at time t_i is f_i and the recovery rate is R
- The loss from defaults at time t_i is $q_i(1-R)E[\max(f_i, 0)]$.
- Defining $u_i = q_i(1-R)$ and v_i as the value of a derivative that provides a payoff of $\max(f_i, 0)$ at time t_i , the cost of defaults is

If Contract Is Always a Liability (equation 22.9)

$$f_0^* = f_0 e^{(y^* - y)T}$$

where derivative provides a payoff at time T .

f_0^* is the actual value of the derivative and

f_0 is the default free value.

y^* is the yield on zero coupon bonds issued by the seller of the derivative and y is the risk-free yield for this maturity

Credit Risk Mitigation

- Netting
- Collateralization
- Downgrade triggers

Default Correlation

- The credit default correlation between two companies is a measure of their tendency to default at about the same time
- Default correlation is important in risk management when analyzing the benefits of credit risk diversification
- It is also important in the valuation of some credit derivatives, eg a first-to-default CDS and CDO tranches.

Measurement

- There is no generally accepted measure of default correlation
- Default correlation is a more complex phenomenon than the correlation between two random variables

Binomial Correlation Measure (page 516)

- One common default correlation measure, between companies i and j is the correlation between
 - A variable that equals 1 if company i defaults between time 0 and time T and zero otherwise
 - A variable that equals 1 if company j defaults between time 0 and time T and zero otherwise
- The value of this measure depends on T . Usually it increases as T increases.

Binomial Correlation continued

Denote $Q_i(T)$ as the probability that company A will default between time zero and time T , and $P_{ij}(T)$ as the probability that both i and j will default. The default correlation measure is

$$\beta_{ij}(T) = \frac{P_{ij}(T) - Q_i(T)Q_j(T)}{\sqrt{[Q_i(T) - Q_i(T)^2][Q_j(T) - Q_j(T)^2]}}$$

Survival Time Correlation

- Define t_i as the time to default for company i and $Q_i(t_i)$ as the probability distribution for t_i
- The default correlation between companies i and j can be defined as the correlation between t_i and t_j
- But this does not uniquely define the joint probability distribution of default times

Gaussian Copula Model

(page 514-515)

- Define a one-to-one correspondence between the time to default, t_i , of company i and a variable x_i by
$$Q_i(t_i) = N(x_i) \quad \text{or} \quad x_i = N^{-1}[Q(t_i)]$$
where N is the cumulative normal distribution function.
- This is a “percentile to percentile” transformation. The p percentile point of the Q_i distribution is transformed to the p percentile point of the x_i distribution. x_i has a standard normal distribution
- We assume that the x_i are multivariate normal. The default correlation measure, ρ_{ij} between companies i and j is the correlation between x_i and x_j

Binomial vs Gaussian Copula Measures (Equation 22.14, page 516)

The measures can be calculated from each other

$$P_{ij}(T) = M[x_i, x_j; \rho_{ij}]$$

so that

$$\rho_{ij}(T) = \frac{M[x_i, x_j; \rho_{ij}] - Q_i(T)Q_j(T)}{\sqrt{[Q_i(T) - Q_i(T)^2][Q_j(T) - Q_j(T)^2]}}$$

where M is the cumulative bivariate normal probability distribution function

Comparison (Example 22.4, page 516)

- The correlation number depends on the correlation metric used
- Suppose $T = 1$, $Q_i(T) = Q_j(T) = 0.01$, a value of ρ_{ij} equal to 0.2 corresponds to a value of $\beta_{ij}(T)$ equal to 0.024.
- In general $\beta_{ij}(T) < \rho_{ij}$ and $\beta_{ij}(T)$ is an increasing function of T

Example of Use of Gaussian Copula (Example 22.3, page 515)

Suppose that we wish to simulate the defaults for n companies . For each company the cumulative probabilities of default during the next 1, 2, 3, 4, and 5 years are 1%, 3%, 6%, 10%, and 15%, respectively

Use of Gaussian Copula continued

- We sample from a multivariate normal distribution to get the x_i
- Critical values of x_i are

$$N^{-1}(0.01) = -2.33, N^{-1}(0.03) = -1.88,$$

$$N^{-1}(0.06) = -1.55, N^{-1}(0.10) = -1.28,$$

$$N^{-1}(0.15) = -1.04$$

Use of Gaussian Copula continued

- When sample for a company is less than -2.33, the company defaults in the first year
- When sample is between -2.33 and -1.88, the company defaults in the second year
- When sample is between -1.88 and -1.55, the company defaults in the third year
- When sample is between -1.55 and -1.28, the company defaults in the fourth year
- When sample is between -1.28 and -1.04, the company defaults during the fifth year
- When sample is greater than -1.04, there is no default during the first five years

A One-Factor Model for the Correlation Structure (Equation 22.10 , page 515)

$$x_i = a_i F + \sqrt{1 - a_i^2} Z_i$$

- The correlation between x_i and x_j is $a_i a_j$
- The i th company defaults by time T when

$$x_i < N^{-1}[Q_i(T)] \text{ or } Z_i < \frac{N^{-1}[Q_i(T) - a_i F]}{\sqrt{1 - a_i^2}}$$

- Conditional on F the probability of this is

$$Q_i(T|F) = N\left\{ \frac{N^{-1}[Q_i(T)] - a_i F}{\sqrt{1 - a_i^2}} \right\}$$

Credit VaR (page 517-519)

- Can be defined analogously to Market Risk VaR
- A T -year credit VaR with an $X\%$ confidence is the loss level that we are $X\%$ confident will not be exceeded over T years

Calculation from a Factor-Based Gaussian Copula Model (equation 22.15, page 517)

- Consider a large portfolio of loans, each of which has a probability of $Q(T)$ of defaulting by time T . Suppose that all pairwise copula correlations are ρ so that all a_i 's are
- We are $X\%$ certain that F is less than $N^{-1}(1-X) = -N^{-1}(X)$
- It follows that the VaR is

$$V(X, T) = N \left\{ \frac{N^{-1}[Q(T)] + \sqrt{\rho} N^{-1}(X)}{\sqrt{1-\rho}} \right\}$$

Basel II

- The internal ratings based approach uses the Gaussian copula model to calculate the 99.9% worst case default rate for a portfolio
- This is multiplied by the loss given default ($=1 - \text{Rec Rate}$), the expected exposure at default, and a maturity adjustment to give the capital required

CreditMetrics (page 517-519)

- Calculates credit VaR by considering possible rating transitions
- A Gaussian copula model is used to define the correlation between the ratings transitions of different companies